Optimized Linear Phase Square-Root Nyquist FIR Filters for CDMA IS-95 and UMTS Standards

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Abstract

In this paper, a novel method to design linear phase square-root Nyquist filters is introduced. The design procedure is posed as a quadratic programming problem with linear and quadratic constraints that can be solved using convex optimization packages. The flatness of the filter’s pass-band i.e., the pass-band ripple energy, is formulated as a quadratic function of the impulse response of the filter. By minimizing this quadratic function through a proxy function called the pseudo-ISI, one can optimize the ISI energy of the filter while the frequency response of the filter satisfies a spectral mask. It is shown that the optimization of the pseudo-ISI results in an optimized ISI energy if the spectral mask is tight enough. The Qualcomm’s Code Division Multiple Access (CDMA) IS-95 and The European Telecommunications Standards Institute’s Universal Mobile Communications Systems (UMTS) standards are used to define the spectral masks. The designed filters show superior inter-symbol interference and stop-band energies compared to the previously proposed designs for CDMA and UMTS standards.

Keywords: Square-root Nyquist Filter, Linear Phase FIR Filters, Inter

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1. Introduction

Square-root Nyquist (SRN) filters are widely used in digital communication systems such as antenna arrays and pulse shaping for different modulation and demodulation schemes. The goal is to design two matching linear phase FIR filters for the transmitter and the receiver in such a way that the cascade impulse response of the filters is a Nyquist pulse shape. If the transfer function of the SRN filters is \( H(z) \) then the cascade transfer function \( G(z) = H(z)H(z^{-1}) \) should satisfy the Nyquist criterion

\[
\sum_{k=0}^{M-1} G(e^{j(\omega + 2\pi k/M)}) = \alpha, \tag{1}
\]

where \( M \) is the over-sampling ratio and \( \alpha \) is a constant number. This criterion is equivalent to the following time-domain Nyquist criterion

\[
g(n) = \begin{cases} 
\alpha/M & n = 0 \\
0 & n = \pm M, \pm 2M, \pm 3M, \cdots 
\end{cases} \tag{2}
\]

where \( g(n) \) is the impulse response of \( G(z) \) and \( g(n) = h(n) \circledast h(-n) \) in which \( h(n) \) is the impulse response of the SRN filters and \( \circledast \) denotes convolution. Obviously, the relationship between \( G(z) \) and \( H(z) \) on the unit circle can be shown as \( |G(e^{j\omega})| = |H(e^{j\omega})|^2 \) or \( |H(e^{j\omega})| = \sqrt{|G(e^{j\omega})|} \) hence the name square-root Nyquist. In designing a SRN filter, several parameters should be considered. These parameters determine the design requirements and making all of them optimum is very challenging and somewhat impossible.
These parameters and their definitions are listed as follows:

1. **Inter Symbol Interference (ISI) Energy**: This, perhaps, the most important parameter in a Nyquist filter. The ISI energy of an ideal filter is zero indicating perfect zero crossing of the cascade impulse response (see (2)) but in many cases where the filter should satisfy a tight spectral mask, zero ISI energy cannot be achieved. We define the ISI energy of a Nyquist filter as

\[ E_i = \sum_{n \in S} |Mg(n) / \alpha|^2 = 2 \sum_{n \in S^+} |Mg(n) / \alpha|^2 \]  

(3)

where \( S = \{ \pm M, \pm 2M, \pm 3M, \cdots \} \), and \( S^+ = \{ M, 2M, 3M, \cdots \} \).

2. **Stop-Band Energy**: The SRN filter is a low-pass filter thus its stop-band energy should be as small as possible. We define the stop-band energy of a SRN filter as

\[ E_s = \frac{1}{\pi} \int_{\omega_s}^{\pi} |H(e^{j\omega})|^2 d\omega, \]

(4)

where \( \omega_s \) is the stop-band corner of the filter.

3. **Tail Energy**: To avoid saturating power amplifiers in communication systems we need to design SRN pulses with reduced peak-to-average ratio (PAR). This can be achieved by reducing the tail energy of the SRN pulses. The tail energy of the pulses \( (E_t) \) can be defined as the energy of the sidelobes of the SRN pulse shape.

4. **Spectral Mask Constraints**: In many practical applications, the SRN filters should satisfy a spectral mask e.g., Code Division Multiple Access (CDMA) IS-95 [1] or Universal Mobile Communications Systems (UMTS) [2] spectral masks. In this paper, we use both spec-
tral masks to design the SRN filters.

The order of the SRN filter is another parameter that needs to be considered. Obviously, the higher the order of the filter is, the higher the cost of its hardware implementation will be. In this paper, we chose a 46\textsuperscript{th} order SRN filter for the CDMA IS-95 standard and a 48\textsuperscript{th} order SRN filter for the UMTS standard. Moreover, we limit our design to linear phase filters in order to utilize their symmetry to lower the hardware implementation costs.

There have been numerous methods offered to design the Nyquist filters. In most of them, a Nyquist filter is designed and then it is factorized to minimum and maximum phase transmitter and receiver filters. These methods are reported in many papers. We will try to cite the papers that have inspired us to come up with the proposed method. Among some of the early methods, we can mention the works of Mueller [3] and Halpern [4] in which the stop band energy of the Nyquist filter was minimized. Liu [5] and Boonyanant [6] used linear programming to design the Nyquist filter. Samueli [7], Farhang-Boroujeny [8] and Boonyanant [6] designed Nyquist filters with equiripple stop band spectrum. A comprehensive study based on the convex optimization was reported by Davidson [9] in which the trade-off curves of the designed Nyquist filters satisfying IS-95 and UMTS proposed standards introduced. Recently, a semi-infinite nonlinear optimization method using dual parameterization approach is proposed by Wu and Teo to design the Nyquist filter [10]. In all of the above methods, the resultant Nyquist filter should be factorized into two SRN filters for the receiver and transmitter. If the Nyquist filter has a positive frequency response, the spectral factorization is possible. The Nyquist filters with equiripple stop band responses can be factorized into minimum and maximum phase filters using
the methods introduced by Herrman [11] and Main [12]. Due to the nature of the factorization process, the resultant transmitter and receiver filters are not linear phase [13], which is the main drawback of the aforementioned methods.

Chevillat [14] and Farhang-Boroujeny [15] introduced methods to design linear phase transmitted and receiver filters. There are two major issues with these two methods. The first problem is the fact that the spectral mask constraints were not included into the optimization algorithms. The second problem is that the ISI energy is a quadratic function of $h(n)$ thus cannot be posed as a convex optimization problem and requires computationally intensive non-convex optimization algorithms. The solution of such algorithms may correspond to a local minimum of the desired cost function.

In this paper, a novel method is introduced to pose the linear-phase SRN filter design as a convex optimization problem in the form of a quadratic programming with linear and quadratic constraints. In this problem, the pseudo-ISI energy of the SRN filter is formulated to quantify the flatness of the pass-band spectrum of the SRN filter. The pseudo-ISI is minimized with the constraints enforcing the spectral mask, prescribed stop-band energy and prescribed tail energy of the SRN filter. The characteristics of the designed filters for the CDMA IS-95 spectral mask are compared with the ones designed by the only similar method available in the literature [15] and it is shown that the proposed method can provide more choices over ISI energy and stop-band energy variations. One of these choices is the optimum SRN filter with the smallest ISI energy. The proposed method is also utilized to design the SRN filters for the UMTS standard. To do so, a spectral mask is proposed that satisfies the frequency response of the available SRN filter for the UMTS (root-raised cosine with excess bandwidth
of 0.22 [2]). The resultant optimum filter shows a significant improvement over the existing UMTS filter.

The organization of the rest of the paper is as follows. In section 2, the problem is formulated and a method to determine the spectral mask that can optimize the ISI energy of the filter is devised. In section 3, the results of numerical simulations are reported and in section 4 the conclusion is drawn.

2. Design Method

2.1. Spectrum

The frequency response of the filter can be given as

$$H(e^{j\omega}) = \sum_{n=0}^{N} h(n) e^{-j\omega n} = e^{-j\omega r} H(\omega)$$

(5)

where $N$ is the order, $h(n)$ is the impulse response and $H(\omega)$ is a real valued function denoting the magnitude response of the filter. By utilizing the even symmetry of the filter, for even values of $N$ we have $r = N/2$, $x \triangleq [x(0), x(1), \cdots, x(r)]^T$ and $c(\omega) \triangleq [1, \text{trig}(\omega, 1), \cdots, \text{trig}(\omega, r)]^T$. On the other hand, for odd values of $N$ we have $r = (N+1)/2$, $x \triangleq [x(1), \cdots, x(r)]^T$ and $c(\omega) \triangleq [\text{trig}(\omega, 1), \cdots, \text{trig}(\omega, r)]^T$. Then we can simplify (5) to

$$H(\omega) = x^T c(\omega),$$

(6)

where $x(n) = 2h(N/2 - n)$ for $1 \leq n \leq r$, $x(0) = 2h(N/2)$ and $\text{trig}(\omega, n) = \cos(\omega n)$ for even values of $N$. Moreover, $x(n) = 2h((N + 1)/2 - n)$ for $1 \leq n \leq r$ and $\text{trig}(\omega, n) = \cos(\omega(n - 1/2))$ for odd values of $N$. The length of $x$ can be determined as $L = r + 1$ when $N$ is even and $L = r$ when $N$ is odd.
2.2. Ripple Energy and pseudo-ISI

Based on (5) and the fact that $H(\omega)$ is a symmetric real-valued function of $\omega$, the ripple energy of the filter can be defined as

$$E_r = \frac{2M}{\pi} \int_0^{\pi/M} \left| \sum_{k=0}^{M-1} H^2(\omega + 2\pi k/M) - \alpha \right|^2 d\omega$$  \hspace{1cm} (7)

The ripple energy $E_r$ shows how much the filter is close to the ideal Nyquist filter according to (1). This quantity is called the spectral flatness in [9]. It should be noted that the summation in (7) is a periodic function of $\omega$ with the period of $2\pi/M$; thus, the integral is taken over one period and multiplied by $M$. If $M$ is not too large ($M \leq 4$), $H^2(\omega + 2\pi k/M)$ is negligible for $\omega \in [0, \pi/M]$ and $1 < k < M - 2$ because they correspond to the stop band region of the filter. This can be seen in Fig. 1 where $H^2(\omega)$ ($k = 1$) and $H^2(\omega - 2\pi k/M)$ ($k = M - 1$) of a SRN filter satisfying CDMA IS-95 spectral mask [15] are obviously the only nonnegligible functions in $\omega \in [0, \pi/M]$. Therefore, we can write

$$E_r \simeq \frac{2M}{\pi} \int_0^{\pi/M} \left| H^2(\omega) + H^2(\omega - 2\pi/M) - \alpha \right|^2 d\omega$$  \hspace{1cm} (8)

If $E_r$ is zero, the spectrum of the Nyquist filter and consequently the spectrum of the SRN filter will be flat and the Nyquist criterion (1) is satisfied, hence zero ISI [16]. Therefore, the ripple energy can be chosen as the cost function to be minimized in order to achieve the minimum ISI energy. It should be noted that the condition $M \leq 4$ is not a necessary condition to obtain (8). We can certainly ignore $H^2(\omega + 2\pi k/M)$ for $\omega \in [0, \pi/M]$ and $1 < k < M - 2$ in (7) when $M > 4$ but the result would not be the most optimized SRN filter satisfying the spectral mask. The condition $M \leq 4$
merely guarantees the true optimality of the result.

Consider the parameters $\omega_p$ and $\omega_s$ as the pass-band and stop-band corners of the SRN filter, respectively. The function $H^2(\omega - 2\pi/M)$ for $\omega \in [0, 2\pi/M - \omega_s]$ and $M = 4$ is also negligible because it corresponds to the stop-band region of the filter (see Fig. 1). The profile of $H^2(\omega - 2\pi/M)$ in the region $\omega \in [2\pi/M - \omega_s, 2\pi/M - \omega_p]$, which corresponds to the transition region of the filter, is dictated by the spectral mask and does not change very much due to the tightness of the mask (it will be assumed later that the mask is tight). Therefore, $H^2(\omega - 2\pi/M)$ will not significantly change with respect to $\mathbf{x}$ while satisfying the mask and will not have much effect on minimizing the ripple energy $E_r$ in the pass band $\omega \in [0, \omega_p]$. Therefore, we can remove it from the cost function (8). The upper limit of the integral (8) should also be changed to $\omega_p$, because the profile of the frequency response of the filter does not change very much within the region $\omega \in [\omega_p, \pi/M]$ due to the tight spectral mask. Consequently, the new cost function is defined as

$$E'_r = \frac{2M}{\pi} \int_0^{\omega_p} |H^2(\omega) - \alpha|^2 d\omega.$$  \hfill (9)

It should be noted that $E'_r > E_r$ because removing $H^2(\omega - 2\pi/M)$ will increase the error $|H^2(\omega) + H^2(\omega - 2\pi/M) - \alpha|$. This can also be seen on Fig. 1. However, the value of the parameter $E'_r$ is not important in the optimization process. The goal is to minimize $E_r$, which can be achieved by minimizing $E'_r$.

Now we define the pass-band spectrum of the SRN filter as

$$H(\omega) = \eta + \beta(\omega), \ \omega \in [0, \omega_p],$$  \hfill (10)
where \( \eta \) and \( \beta(\omega) \) are the average gain and the ripple function of the filter in \( \omega \in [0, \omega_p] \), respectively. By substituting (10) into (9) and expanding the equation, we can obtain:

\[
E'_r = \frac{2M}{\pi} \int_{0}^{\omega_p} \left| \beta^2(\omega) + \eta^2 + 2\eta\beta(\omega) - \alpha \right|^2 d\omega.
\]  

(11)

The function \( \beta(\omega) \) represents the ripple of the pass band and \( \eta \simeq 1 \). We can assume that the peak-to-peak variations of \( \beta(\omega) \) are very small, i.e. \( |\beta(\omega)|^2 \ll |\beta(\omega)| < 1 \). Therefore, by substituting \( \beta(\omega) \) in (11) with \( H(\omega) - \eta \) from (10), ignoring \( \beta^2(\omega) \) and performing some simple algebraic manipulations, we can easily obtain

\[
E'_r \simeq \frac{8M\eta^2}{\pi} \int_{0}^{\omega_p} \left| H(\omega) - \frac{\eta^2 + \alpha}{2\eta} \right|^2 d\omega.
\]  

(12)

By defining \( \delta = (\eta^2 + \alpha)/2\eta \) and

\[
E'_r = \frac{8M\eta^2 E_p}{\pi},
\]  

(13)

we can introduce the parameter \( E_p \)

\[
E_p = \int_{0}^{\omega_p} \left| H(\omega) - \delta \right|^2 d\omega.
\]  

(14)

If the new quantity \( E_p \) is minimized, the ripple energy and consequently the ISI energy of the filter is minimized. Therefore, we call \( E_p \) the pseudo-ISI of the filter. By using (6), we can easily present \( E_p \) in a quadratic form

\[
E_p = \hat{x}^T Q_p \hat{x},
\]  

(15)
where \( \hat{x} = [x^T, \delta]^T \),

\[
Q_p = \begin{bmatrix}
Q_c & u \\
0 & \omega_p
\end{bmatrix}, \quad Q_p \in \mathbb{R}^{(L+1) \times (L+1)},
\]  

(16)

and

\[
Q_c = \int_{\omega_p}^{\omega_p} c(\omega)c(\omega)^T d\omega, \quad u = -\int_{0}^{\omega_p} c(\omega)^T d\omega.
\]  

(17)

It should be mentioned that \( \delta \) is merely an auxiliary parameter in the optimization and its optimized value is used in determining the SRN filter coefficients.

2.3. Stop-band Energy

According to the definition of the frequency response of the SRN filter (6), the stop-band energy of the filter can be shown as

\[
E_s = \frac{1}{\pi} \int_{\omega_s}^{\pi} |H(e^{j\omega})|^2 d\omega = \hat{x}^T Q_s \hat{x},
\]  

(18)

where \( Q_s \in \mathbb{R}^{(L+1) \times (L+1)} \) is defined as

\[
Q_s = \begin{bmatrix}
Q & 0 \\
0 & 0
\end{bmatrix}, \quad Q = \frac{1}{\pi} \int_{\omega_s}^{\pi} c(\omega)c(\omega)^T d\omega.
\]  

(19)

2.4. Tail Energy

It is also not difficult to show that the tail energy of the filter is given by

\[
E_t = \hat{x}^T Q_t \hat{x},
\]  

(20)
where $Q_t \in \mathbb{R}^{(L+2) \times (L+2)}$ is a diagonal matrix taking into account the samples of the side-lobes. The matrix $Q_t$ can be shown as

$$Q_t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

(21)

where $I \in \mathbb{R}^{(L-M+2) \times (L-M+2)}$ is an identity matrix.

Now we have all the necessary parameters as convex functions of $x$, thus we can formulate the optimization problem to design the SRN filter.

2.5. Problem Formulation

The goal is to minimize the pseudo-ISI with the given constraints. Therefore, the design problem can be written as the following quadratic problem with linear and quadratic constraints

$$\min \hat{x}^T Q_p \hat{x},$$

subject to

$$\hat{x}^T Q_s \hat{x} \leq \xi_s,$$

$$\hat{x}^T Q_t \hat{x} \leq \xi_t,$$

$$M_l(e^{j\omega}) \leq H(\omega) \leq M_u(e^{j\omega}),$$

(22)

where $M_l(e^{j\omega})$ and $M_u(e^{j\omega})$ determine the spectral mask, $\xi_s$ is the stop-band energy bound and $\xi_t$ is the tail energy bound of the filter. The spectral mask constraint is semi-infinite, i.e., it creates infinite linear constraints. This semi-infinite constraint can be transformed to a finite set of constraints by discretizing the frequency variable $\omega$ over a sufficiently fine grid. Usually we choose $15N$ uniformly spaced frequency points plus the frequency corners of the mask to create the finite number of constraints [9]. Then the problem
(22) with finite set of spectral constraints can be easily solved by convex or quadratic optimization software packages such as CVX [17] or IBM ILOG CPLEX Optimizer.

2.6. Finding the Spectral Mask

The validity of the proposed method depends on the spectral mask tightness condition. As mentioned before, this tightness is required to make sure that the profile of the filter’s frequency response does not change with the variation of the vector \( x \) in the optimization setting (22). The tightness of the spectral mask can be guaranteed by reducing the pass-band ripple dynamic range. We can tighten the spectral mask by other means including reducing the width of the transition band but reducing the pass-band dynamic range can also help us in minimizing the ripple energy and consequently the ISI energy.

Finding a tight enough spectral mask depends on the targeted characteristics of the filter. In this process, the designer should avoid over-tightening the spectral mask because it could cause the optimization problem (22) become infeasible. On the other hand making a loose spectral mask will cause inaccurate results. In general, a tight enough spectral mask can be found using the following procedure:

1. A spectral mask should be chosen (or it may be given as part of the design requirements). This mask should be considered as the loosest mask allowed in this process.

2. The maximum possible stop-band energy of the filter for the given mask should be determined by solving (22) where \( \xi_s \) is set to be a large number.
3. The SRN filter should be designed for descending values of the stop-band energy starting from the maximum stop-band energy found in step-2. The smallest value of the stop-band energy can be easily identified in this iteration when the optimization problem (22) becomes infeasible.

4. The trade-off curve (ISI energy versus stop-band energy) of the SRN filter is plotted.

5. The spectral mask should be tightened and the above iteration should be repeated. The optimum trade-off curve is the closest to the origin.

This procedure is used to design the SRN filter for the CDMA IS-95 and UMTS standards, which is explained in the next section. The above procedure is depicted in Fig. 2.

3. Simulation Results

3.1. The CDMA IS-95 Filter

3.1.1. Choosing the Spectral Mask

According to the CDMA IS-95 standard [1, p. 6-29], the spectral mask for the optimization problem (22) can be written as

\[ M_l(e^{j\omega}) = \begin{cases} 
10^{-1.5/20} & \omega \leq \omega_p \\
-10^{-40/20} & \omega \geq \omega_s 
\end{cases} \]  

(23)

\[ M_u(e^{j\omega}) = \begin{cases} 
10^{1.5/20} & \omega \leq \omega_p \\
10^{-40/20} & \omega \geq \omega_s 
\end{cases} \]  

(24)

where \( \omega_s \) and \( \omega_p \) are the stop-band and pass-band corners of the spectral mask, respectively [1]. The spectral mask defined by (23) and (24) indicate a
−40dB stop-band gain and a pass-band dynamic range of $\sigma = \pm 1.5$ dB. We can tighten the spectral mask by reducing the pass-band dynamic range $\sigma$. Fig. 3 illustrates the trade-off curves obtained by solving (22) with the given parameters for different values of the pass-band dynamic range $\sigma$. It can be seen from the figure that the best spectral mask corresponds to $\sigma = 1.5$ dB, which is the original mask given by the CDMA IS-95 standard.

3.1.2. The effect of $\xi_t$

Since the restriction on the frequency response of the SRN filter imposed by the spectral mask is too tight, we cannot arbitrarily change the tail energy of the filter. Therefore, too small values for $\xi_t$ will make (22) infeasible. On the other hand, reducing the tail energy causes higher ISI and stop-band energies. Fig. 4 illustrates the eye diagram for $\xi_t = \infty$ and $\xi_t = 0.0102$. Obviously, the eye diagram is flattened by reducing the tail energy but its ISI and stop-band energies are also increased. The optimized values of the auxiliary parameter $\delta$ are respectively 1.0171 and 1.0443 for the aforementioned designs.

3.1.3. Comparisons

To have a general comparison, the trade-off curve of the SRN design proposed in this paper, the trade-off curve of the design introduced in [15] (with $M = 4$ to obtain the same stop band corner) and the original IS-95 filter reported in [1] are compared in Fig. 5. Obviously, the proposed method can provide the SRN filters with better ISI and stop-band energies compared to the original CDMA IS-95 filter and the filters obtained by the method given in [15]. The optimum filter is the one whose corresponding point on the curve shown in Fig. 5 has the smallest ISI energy. This filter has the
stop-band energy of $3.49 \times 10^{-5}$, the ISI energy of $1.35 \times 10^{-3}$ (which is the smallest ISI energy for a CDMA IS-95 SRN filter reported in the literature) and $\delta = 1.0061$. Moreover, the proposed method offers a wider range of ISI energy versus stop-band energy for the SRN filter as opposed to the method discussed in [15]. It should be noted that the filter given in [15] has a better stop-band energy compared to the optimum filter. However, since the filters all satisfy the CDMA IS-95 spectral mask, the higher stop-band energy would be of no concern in the design of the communication system in terms of the adjacent channel interference.

Fig. 6 illustrates the frequency response of the optimum SRN filter satisfying the CDMA IS-95 spectral mask (introduced above) along with the CDMA IS-95 filter (ISI energy of $22.6 \times 10^{-3}$ and the stop-band energy of $1.6 \times 10^{-4}$) and the filter proposed in [15] (ISI energy of $3.7 \times 10^{-3}$ and the stop-band energy of $2.2 \times 10^{-6}$). It can be seen from Fig. 6 that the pass-band ripples of the optimum filter are smaller than those of the CDMA IS-95 filter and the filter given in [15]. This confirms the theoretical analysis given in 2.2 that is based on the minimization of the pass-band ripple or pseudo-ISI.

Another method that was introduced by Davidson can also be used to design SRN filters satisfying the CDMA IS-95 spectral mask [9]. This method designs the optimum Nyquist filter and then uses spectral factorization to create the SRN matching filters. The main disadvantage of this method is the fact that the final SRN filters are nonlinear phase (nonsymmetric). Therefore, they have twice the number of coefficients, compared to the proposed linear phase filter, to be realized in hardware hence more complex system and larger silicon area.
3.2. The UMTS Filter

3.2.1. Choosing the Spectral Mask

The UMTS standard does not provide a spectral mask for the pulse shaping filter. Instead, a root raised-cosine filter with the excess bandwidth (roll-off factor) of 0.22 is used as the pulse shaping filter. To design a more efficient SRN filter, we can define a spectral mask in which the root raised-cosine filter with $N = 48$ and $M = 4$ fits. This method was first introduced by Davidson in [9], where the formula of the spectral mask is defined as

$$M_l(e^{j\omega}) = \begin{cases} 
10^{-\sigma} & \omega \leq \omega_p \\
-10^{A_{s1}} & \omega_{s1} \leq \omega \leq \omega_{s2} \\
-10^{[\frac{(A_{s2}-A_{s1})}{\omega_{s3}-\omega_{s2}}+A_{s1}]} & \omega_{s2} \leq \omega \leq \omega_{s3} \\
-10^{A_{s2}} & \omega \geq \omega_{s3}
\end{cases}$$

$$M_u(e^{j\omega}) = \begin{cases} 
10^\sigma & \omega \leq \omega_{s1} \\
10^{A_{s1}} & \omega_{s1} \leq \omega \leq \omega_{s2} \\
10^{[\frac{(A_{s2}-A_{s1})}{\omega_{s3}-\omega_{s2}}+A_{s1}]} & \omega_{s2} \leq \omega \leq \omega_{s3} \\
10^{A_{s2}} & \omega \geq \omega_{s3}
\end{cases}$$

(25)

where $\omega_p$, $\omega_{s1}$, $\omega_{s2}$, $\omega_{s3}$, $\sigma$, $A_{s1}$, and $A_{s2}$ are the pass-band corner, first stop-band corner, second stop-band corner, third stop-band corner, pass-band dynamic range, first stop-band gain and second stop-band gain, respectively. Moreover, the values of these parameters used in [9] are given in Table 1. Fig. 7 shows this spectral mask with the frequency response of the root raised cosine filter used in the UMTS standard.

3.2.2. The Trade-off Curves

The spectral mask chosen in [9] is too loose. To gradually tighten the mask, we consider five different values for $\omega_p$ and $\sigma$ along with the frequency
responses of the root raised-cosine filter to create five different spectral masks (the first mask is the one given in Table 1). The trade-off curves of these five masks obtained by solving (22) are depicted in Fig. 8. As it can be seen in Fig. 8, the tightest mask with $\sigma = 0.1$ and $f_p = 0.09896$ provides the optimum SRN filter with the stop-band energy of $3.605 \times 10^{-6}$ and the ISI energy of $2.147 \times 10^{-6}$ which are far better than the corresponding values of the UMTS root raised-cosine filter (shown by “×” in the figure). The frequency response of this optimum filter is shown in Fig. 9, where one can see significant reduction of the pass-band ripple magnitude, hence very low ISI.

It should be noted that the obtained filter is a linear-phase FIR SRN filter while the filter obtained by Davidson in [9] is a minimum phase SRN filter but it is not linear-phase. Therefore the hardware implementation of the Davidson’s filter is twice costly than that of the optimum filter designed in this paper. To the best of our knowledge, this is the most efficient linear-phase FIR filter for the UMTS standard that has been proposed in the literature. In the above computations, the effect of $\xi_t$ is ignored, i.e., $\xi_t = \infty$.

3.2.3. The effect of $\xi_t$

Since the spectral mask is chosen to be very tight, the parameter $\xi_t$ can be selected from a very narrow interval (between 0.011 to 0.0132) and has insignificant effect on the tail energy. In order to widen the interval from which $\xi_t$ can be selected, we should loosen the spectral mask, which in turn deteriorate the ISI and stop-band energy of the filter.
4. Conclusions

A new method to design linear phase SRN FIR filter is presented in this paper. The ISI energy of the filter is optimized by minimizing the ripple energy of the pass-band of the filter. The design method is posed as a quadratic programming with linear and quadratic constraints. The trade-off curves of the designed filters are calculated, which show the effectiveness of the design method compared to the previously reported algorithms. By including the constraint of the tail energy of the filter, one can somewhat reduce the tail energy and consequently decrease the peak-to-average ratio of the filter. However, the tightness of the spectral mask prevents arbitrary reduction of the tail energy. The proposed method is employed to design the optimum SRN filter satisfying the CDMA IS-95 spectral mask. Moreover, the proposed method is used to design the optimum SRN filter suitable for the UMTS standard. To the best of our knowledge, the designed optimum filters are the most efficient SRN filter for the CDMA IS-95 and UMTS standards in the literature.


Table 1: Parameters of the UMTS spectral mask \[9\].

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<th>Parameter</th>
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<tbody>
<tr>
<td>$\omega_p$</td>
<td>$2\pi \times 1.2$</td>
</tr>
<tr>
<td>$\omega_{s1}$</td>
<td>$2\pi \times 0.156$</td>
</tr>
<tr>
<td>$\omega_{s2}$</td>
<td>$2\pi \times 0.168$</td>
</tr>
<tr>
<td>$\omega_{s3}$</td>
<td>$2\pi \times 0.3274$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2 dB</td>
</tr>
<tr>
<td>$A_{s1}$</td>
<td>−34 dB</td>
</tr>
<tr>
<td>$A_{s2}$</td>
<td>−59.5 dB</td>
</tr>
</tbody>
</table>
Figure 1: The magnitude response $H^2(\omega)$ of a square Nyquist filter satisfying the CDMA-I95 spectral mask with $M = 4$ [15] and its replicas $H^2(\omega + 2\pi k/M)$. 
Choosing the initial spectral mask (The loosest mask)

Find the maximum possible stop-band energy for the given mask by solving (22)

Is optimization problem (22) feasible?

Yes

Decrease the stop-band energy and use (22) to design the filter

Is optimization problem (22) infeasible?

No

Plot the trade-off curve

Tighten the mask

No

End

Figure 2: The flowchart of the procedure to find the tight enough spectral mask.
Figure 3: The trade-off curves for different values of the pass-band dynamic range $\sigma$ for the CDMA IS-95 filter.
Figure 4: The eye diagrams of the SRN filters satisfying the CDMA IS-95 spectral mask
(a) The optimum SRN filter with the tail energy of 0.0167. (b) The SRN filter with minimum possible tail energy of 0.0102 (stop-band energy is $1.65 \times 10^{-5}$ and the ISI energy is $4.2 \times 10^{-2}$).
Figure 5: The comparison curves for the SRN filters satisfying the CDMA IS-95 spectral mask.
Figure 6: The magnitude response of the optimum SRN filter satisfying the CDMA IS-95 spectral mask, CDMA IS-95 SRN filter and the filter introduced in [15].
Figure 7: The UMTS spectral mask proposed in [9] (dotted line) and the root raised-cosine filter used in [2] (solid line).
Figure 8: The trade-off curves of the SRN filters satisfying the UMTS spectral mask for different values of the pass-band gain $\sigma$ and corner $f_p$. 
Figure 9: The frequency response of the UMTS optimum filter compared with the UMTS root raised-cosine filter. The dotted line shows the chosen spectral mask for $\sigma = 0.1$ dB and $f_p = 0.09896$. 